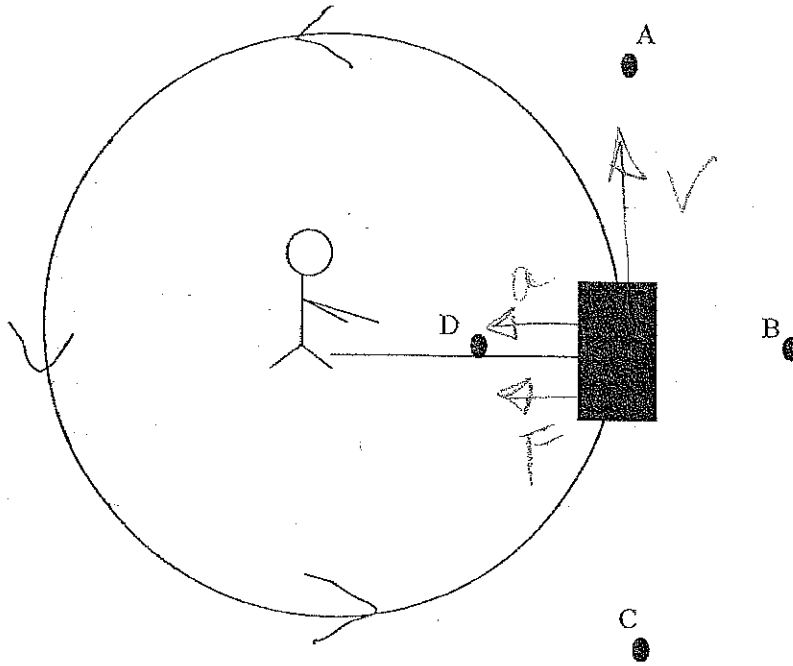


Diagram of Uniform Circular Motion

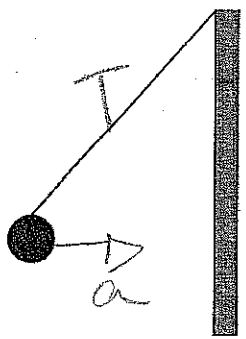
- Label the velocity vector for the object
- Label the acceleration vector of the object
- Label the force vector of the object
- Predict the motion of the object if the string is cut at the position shown



Go to A!

Tetherball (Conical Pendulum)

Jason hits a tetherball around a pole... For fun.
In what direction is the acceleration of the ball and what causes this acceleration?



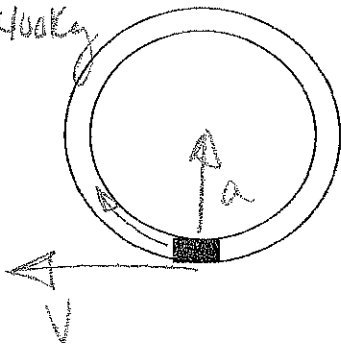
towards the pole
 T_x causes accel
 $\therefore T_x = F_c$

Uniform Circular Motion Problems

1. Zack is driving a 2,400 kg race car and is traveling around a circular track that has a radius of 35 meters. It takes him 23.2 seconds to complete one lap,

- What is the speed with which Zack and the car make it around the track?
- What is the acceleration?
- What is the centripetal force?
- What force is the cause of this circular motion?
- On the diagram below, label a vector to indicate the centripetal acceleration of the car at the time shown, and label a vector to indicate the velocity of the car at the time shown. The car is traveling in a clockwise direction.

$r = 35\text{m}$
 $T = 23.2\text{s}$
 $m = 2,400\text{kg}$



$$v = \frac{2\pi r}{T} = \frac{2\pi(35\text{m})}{23.2\text{s}} = 9.5\text{m/s}$$

$$b) a = \frac{v^2}{r} = \frac{(9.5\text{m/s})^2}{35\text{m}} = 2.6\text{m/s}^2$$

$$c) F = ma_c = 2400\text{kg}(2.6\text{m/s}^2) = 6240\text{N}$$

d) Friction tires on road

2. John whirls a 7.00 kg hammer tied to the end of a 1.3 m chain in a horizontal circle. The hammer makes one revolution in 1.0 s at a constant speed.

- What is the speed of the hammer around John?
- What is the centripetal acceleration of the hammer?
- What is the magnitude of centripetal force acting on this hammer?
- What force accelerates the hammer?

$r = 1.3\text{m}$
 $T = 1\text{s}$
 $m = 7\text{kg}$

$$a) v = \frac{2\pi r}{T} = \frac{2\pi(1.3\text{m})}{1\text{s}} = 8.2\text{m/s}$$

$$c) F_c = ma_c = 7\text{kg}(57.7\text{m/s}^2) = 361.9\text{N} = 362\text{N}$$

$$b) a_c = \frac{v^2}{r} = \frac{(8.2\text{m/s})^2}{1.3\text{m}} = 51.7\text{m/s}^2$$

d) Tension in the chain

3. Maddy is riding on the merry-go-round at Carousel Mall and is moving with a speed of 1.00 m/s when she is 1.08 m from the center of the merry-go-round.

- Calculate Maddy's centripetal acceleration.
- Calculate the net horizontal force exerted on Maddy (mass = 40.0 kg).
- Calculate the frequency of revolution Maddy has on the merry-go-round.

$r = 1.08\text{m}$
 $v = 1\text{m/s}$

$$a) a_c = \frac{v^2}{r} = \frac{(1\text{m/s})^2}{1.08\text{m}} = 0.93\text{m/s}^2$$

$$c) v = \frac{2\pi r}{T}$$

$$1 = \frac{2\pi(1.08)}{T}$$

$$b) F_c = ma_c = 40\text{kg}(0.93\text{m/s}^2) = 37\text{N}$$

$$T = 6.79\text{s}$$

$$f = \frac{1}{T} = 0.14\text{s}^{-1} \text{ or } \text{Hz}$$

4. Jake has tied a 1.8 kg stone to one end of a string and whirls it at a constant speed of 2.5 m/s along a horizontal circle of radius 1.3 m. Determine:

- the time period for the circular motion
- the frequency of the circular motion
- the centripetal acceleration
- the tension in the string

$$r = 1.3 \text{ m}$$

$$m = 1.8 \text{ kg}$$

$$v = 2.5 \text{ m/s}$$

a) $v = \frac{2\pi r}{T}$
 $2.5 \text{ m/s} = \frac{2\pi(1.3 \text{ m})}{T}$
 $T = 3.27 \text{ s}$

b) $f = \frac{1}{T}$
 $f = \frac{1}{3.27 \text{ s}}$
 $f = 0.31 \text{ s}^{-1}$
 or 0.31 Hz

c) $a_c = \frac{v^2}{r}$
 $= \frac{(2.5 \text{ m/s})^2}{1.3 \text{ m}}$
 $a_c = 4.8 \text{ m/s}^2$

d) $F_T = F_c$
 $= mac$
 $= 1.8 \text{ kg}(4.8 \text{ m/s}^2)$
 $= 8.7 \text{ N}$

5. A 0.40 kg soccer ball, attached to the end of a horizontal cord, is being rotated by Tom in a circle of radius 1.5 m on the (nearly) frictionless gym (frictionless for our purposes) floor, horizontally. If the cord will break when the tension in it exceeds 65 N, what is the maximum speed the ball can have?

$$m = 0.4 \text{ kg}$$

$$r = 1.5 \text{ m}$$

$$T > 65 \text{ N, breaks}$$

$$v = ?$$

$$T = F_c = \frac{mv^2}{r}$$

$$65 \text{ N} = \frac{0.4 \text{ kg} v^2}{1.5 \text{ m}}$$

$$v = \sqrt{\frac{(1.5)(65)}{0.4}} = 15.6 \text{ m/s}$$

6. What would happen to centripetal force if:

- The mass of the object was tripled? $F_c = \frac{mv^2}{r}$ $3 \cdot \frac{1^2}{1} = 3 \times F_c$
- The radius of the object was doubled? $\frac{1 \cdot 1^2}{2} = \frac{1}{2} F_c$
- The velocity of the object was doubled? $\frac{1 \cdot 2^2}{1} = 4 \times F_c$
- The mass was halved and the radius was quadrupled? $\frac{0.5(1)^2}{4} = 0.125 F_c$ or $\frac{1}{8} F$
- The period of the object was doubled?

$$v = \frac{2\pi r}{T} = \frac{1 \cdot 1 \cdot 1}{2} \quad v = \frac{1}{2} \quad \frac{1 \cdot (\frac{1}{2})^2}{1} = \frac{1}{4} F_c$$

Horizontal Circular Motion Problems

1. Alex is driving his car and is traveling on a 20 m radius turn. If the coefficient of static friction between the tires and the surface of the road is $\mu_s = 0.6$, determine the maximum possible safe speed Alex can travel at to avoid slipping.

$$r = 20\text{m}$$

$$\mu_s = 0.6$$

$$v = ?$$

$$F_c = F_f$$

$$\frac{mv^2}{r} = \mu FN$$

$$\frac{mv^2}{r} = \mu mg$$

$$v = \sqrt{\mu g r}$$

$$= \sqrt{0.6(9.8\text{m/s}^2)(20\text{m})}$$

$$= 10.8\text{m/s}$$

2. a. What is the maximum speed with which a 1180 kg car can round a turn of radius 80 m on a flat road if the coefficient of friction between tires and road is 0.80?

b. Is this result independent of the mass of the car? *yes*

mass cancels!

$$v = ?$$

$$r = 80\text{m}$$

$$\mu_s = 0.8$$

$$m = 1180\text{kg}$$

$$F_c = F_f$$

$$\frac{mv^2}{r} = \mu FN$$

$$\frac{mv^2}{r} = \mu mg$$

$$v = \sqrt{\mu g r}$$

$$v = \sqrt{0.8(9.8)(80)}$$

$$v = 25\text{m/s}$$

3. How large must the coefficient of static friction be between the tires and the road if a car is to round a level curve of radius 76 m at a speed of 90 km/h?

$$\mu = ?$$

$$r = 76\text{m}$$

$$v = 90 \frac{\text{km}}{\text{h}} \times \frac{1000\text{m}}{1\text{km}} \times \frac{1\text{hr}}{3600\text{s}} = 25\text{m/s}$$

$$F_c = F_f$$

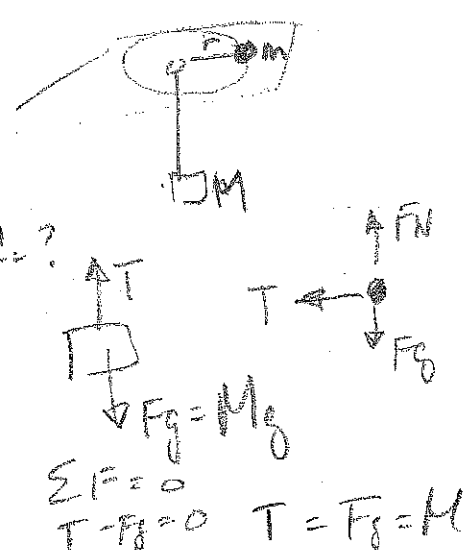
$$\frac{mv^2}{r} = \mu FN$$

$$\frac{mv^2}{r} = \mu mg$$

$$\mu = \frac{v^2}{r g}$$

$$\mu = 0.84$$

4. A string passing through a hole in a table carries a mass m at one end and a mass M at the other end. Mass m is placed on a horizontal frictionless table and mass M is hanging freely. The length of the string from the hole to the mass m is r . Determine the frequency with which the mass m must move in a circle of radius r so that the mass M stays at rest.



$$T = F_c$$

$$Mg = \frac{mv^2}{r}$$

$$v = \sqrt{\frac{Mgr}{m}}$$

$$v = \frac{2\pi r}{T}$$

$$v = \frac{2\pi r}{\frac{1}{f}}$$

$$v = 2\pi r f$$

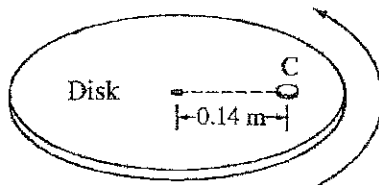
$$\frac{\sqrt{\frac{Mgr}{m}}}{2\pi r} = \frac{2\pi r f}{2\pi r}$$

$$f = \sqrt{\frac{Mg}{m}}$$

$$f = \sqrt{\frac{Mg}{4\pi^2 m r}}$$

$\Sigma F = 0$
 $T - F_g = 0$
 $T = F_g = Mg$

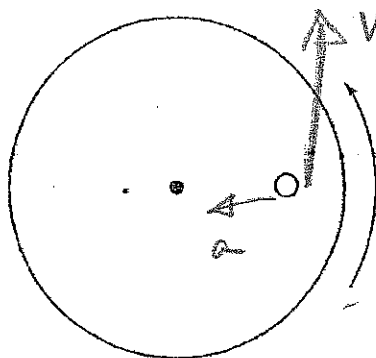
Practice AP Problem



A coin C of mass 0.0050 kg is placed on a horizontal disk at a distance of 0.14 m from the center, as shown above. The disk rotates at a constant rate in a counterclockwise direction as seen from above. The coin does not slip, and the time it takes for the coin to make a complete revolution is 1.5 s.

- a. The figure below shows the disk and coin as viewed from above. Draw and label vectors on the figure below to show the instantaneous acceleration and linear velocity vectors for the coin when it is at the position shown.

$m = 0.005 \text{ kg}$
 $r = 0.14 \text{ m}$
 $T = 1.5 \text{ s}$



- b. Determine the linear speed of the coin.

$$v = \frac{2\pi r}{T} = \frac{2\pi (0.14 \text{ m})}{1.5 \text{ s}} = 0.59 \text{ m/s}$$

- c. The rate of rotation of the disk is gradually increased. The coefficient of static friction between the coin and the disk is 0.50. Determine the linear speed of the coin when it just begins to slip.

$\mu = 0.5$

$F_c = F_f$

$v = \sqrt{\mu g r}$

$m v^2 = \mu F_N$

$v = \sqrt{0.5 (9.8 \text{ m/s}^2) (0.14 \text{ m})}$

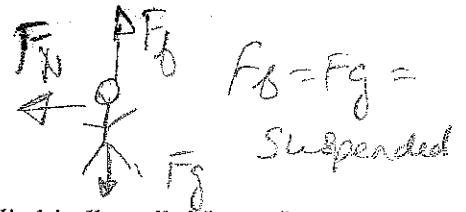
$\frac{m v^2}{r} = \mu m g$

$v = 0.83 \text{ m/s}$

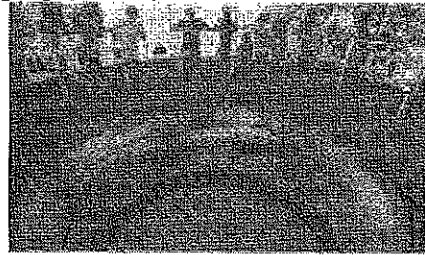
- d. If the experiment in part (c) were repeated with a second, identical coin glued to the top of the first coin, how would this affect the answer to part (c)? Explain your reasoning.

Mass cancels, so ~~the~~ mass would have no effect

Rotor Circular Motion Problems



1. In a "Rotor-ride" at a carnival, people pay money to be rotated in a vertical cylindrical walled "room."



Normal Force holds you against the wall not friction!

a. If the room radius is 4.7 m, and the rotation frequency is 0.7 revolutions per second when the floor drops out, what is the minimum coefficient of static friction so that the people will not slip down?

$r = 4.7 \text{ m}$
 $f = 0.7 \text{ Hz}$
 $T = 1.43 \text{ s}$

$v = 2\pi r f$
 $F_c = F_N$
 $\frac{mv^2}{r} = \frac{mg}{\mu}$
 $F_b = F_g$
 $\mu F_N = mg$
 $\mu = \frac{g r}{(2\pi r f)^2} = \frac{g}{4\pi^2 f^2 r}$
 $\mu = \frac{9.8}{4\pi^2 (0.7)^2 (4.7)} = 0.11$

b. People describe this ride by saying they were being "pressed against the wall." Is this true? Is there really an outward force pressing them against the wall?

No, wall is pushing them in. They are the outward force on the wall.

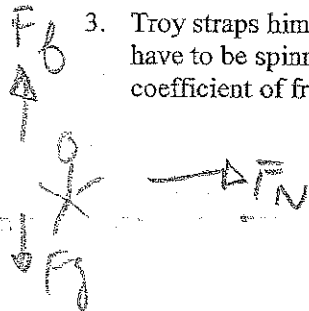
$\mu = 0.11$

2. A rotor in an amusement park has a radius of 2.3 m. The coefficient of static friction between Adam who is the rider and the rotor wall is 0.5. Determine the minimum frequency of the rotor above which Adam will not slip down when the floor of the rotor drops.

$r = 2.3 \text{ m}$
 $\mu = 0.5$
 $f = ?$

$v = 2\pi r f$
 $\frac{mv^2}{r} = \frac{mg}{\mu}$
 $\mu \frac{m(2\pi r f)^2}{r} = mg$
 $4\pi^2 r^2 f^2 \mu = r g$
 $4\pi^2 (2.3 \text{ m})^2 f^2 (0.5) = 2.3 \text{ m} (9.8 \text{ m/s}^2)$
 $f = 0.46 \text{ Hz}$

3. Troy straps himself into a 5.0 m radius Gravitron ride at the local fair. At what frequency does the Gravitron have to be spinning, in order for a 80. kg Troy to stay suspended (at rest) during the duration of the ride. The coefficient of friction between Troy and the ride is 0.4.



$F_b = F_g$
 $\mu F_N = mg$
 $F_N = \frac{mg}{\mu}$

$F_N = F_c$
 $\frac{mg}{\mu} = \frac{mv^2}{r}$

$\frac{gr}{\mu} = v^2$
 $v = \frac{9.8 \text{ m/s}^2 (5 \text{ m})}{0.4}$

$v = 11.75 \text{ m/s}$

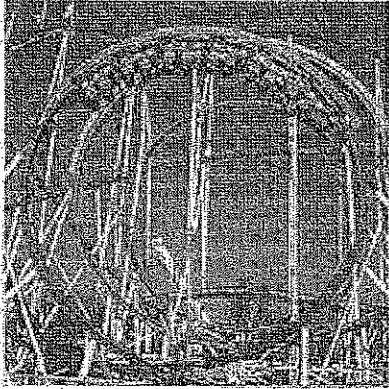
$v = 2\pi r f$
 $11.75 = 2\pi (5 \text{ m}) f$
 $f = 0.35 \text{ Hz}$

$r = 5 \text{ m}$
 $m = 80 \text{ kg}$
 $\mu = 0.4$

$f = 0.46 \text{ Hz}$

Vertical Circle Problems

1. At what minimum speed must a roller coaster be traveling when upside down at the top of a circle (Fig. 5-34) if the passengers are not to fall out? Assume a radius of curvature of 8.4 m.



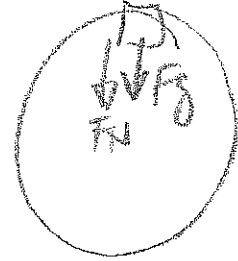
$$F_c = F_N + F_g$$

$$\frac{mv^2}{r} = mg$$

$$v = \sqrt{gr}$$

$$v = \sqrt{(9.8 \text{ m/s}^2)(8.4 \text{ m})}$$

$$v = 9.1 \text{ m/s}$$



$$\Sigma F = F_N + F_g$$

$$F_c = F_N + F_g$$

$$v = ?$$

$$r = 8.4 \text{ m}$$

2. Alexis decides to be a daredevil motorcyclist and chooses to drive around a loop de loop track with a radius of 5.2 meters. At what minimum speed must she ride in the loop to loop so that she does not fall to her doom?

$$r = 5.2 \text{ m}$$

$$v = ?$$

$$F_c = F_N + F_g$$

$$\frac{mv^2}{r} = mg$$

$$v = \sqrt{gr}$$

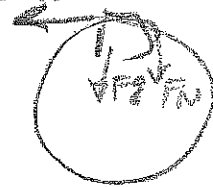
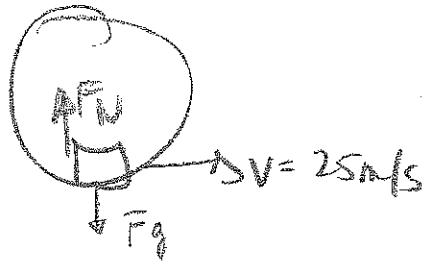
$$v = \sqrt{(9.8 \text{ m/s}^2)(5.2 \text{ m})}$$

$$v = 7.14 \text{ m/s}$$

3. A loop de loop rollercoaster has a radius of 30 meters. Determine the apparent weight a 500 N person will feel at the bottom of the loop, traveling at a speed of 25 m/s and at the top of the loop, traveling at a speed of 20 m/s.

= 51 kg

$r = 30m$



$$F_c = F_N - F_g$$

$$\frac{mv^2}{r} = F_N - mg$$

$$\frac{51kg (25m/s)^2}{30m} = F_N - 500N$$

$$F_N = 1062.5 + 500 = \boxed{1562.5N} \approx 3.125g's!$$

$$F_c = F_N + mg$$

$$\frac{mv^2}{r} = F_N + 500N$$

$$\frac{51kg (20m/s)^2}{30m} = 500N = F_N$$

$$\boxed{F_N = 1500N}$$

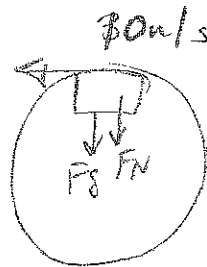
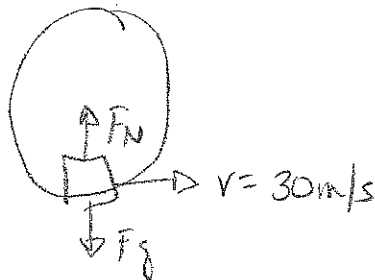
4. A loop-de-loop rollercoaster has a radius of 25 m. Determine the apparent weight a 600 N rider would feel:
 a. traveling at a speed of 30 m/s at the bottom of the vertical arc
 b. traveling at a speed of 10m/s at the top of the vertical arc.

$r = 25m$

$F_g = 600N$

$F_N = ?$

$m = 61.2kg$



0.36g's!

$$F_c = F_N - F_g$$

$$\frac{mv^2}{r} = F_N - mg$$

$$F_N = \frac{(61.2kg)(30m/s)^2}{25m} + 600N$$

$F_N = 2803N$
 (4.6g's)

$$F_c = F_N + F_g$$

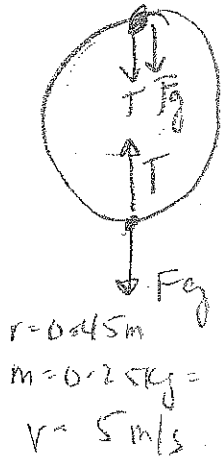
$$\frac{mv^2}{r} = F_N + mg$$

$$F_N = \frac{61.2kg(10m/s)^2}{25m} - 600N$$

∴ $F_N = -355N$ - can't make it around the loop!
 not going fast enough

5. At the 2009 International Yoyo contest, a Yoyo enthusiast Evan Evans spins a 0.25 kg yoyo in a vertical trajectory. The radius of this path is 45 centimeters and he spins it at a constant speed of 5.0 m/s.

- What is the tension of the string when the yoyo is at the top of the path?
- What is the tension of the string at the bottom of the path?
- Where in the yoyo's path is the string most likely to break, if it is spun too fast?



$$a) F_c = T + F_g$$

$$\frac{mv^2}{r} = T + mg$$

$$\frac{(0.25 \text{ kg})(5 \text{ m/s})^2}{0.45 \text{ m}} = T + 0.25 \text{ kg}(9.8 \text{ m/s}^2)$$

$$T = 11.4 \text{ N}$$

$$b) F_c = T - F_g$$

$$\frac{mv^2}{r} = T - mg$$

$$\frac{(0.25 \text{ kg})(5 \text{ m/s})^2}{0.45 \text{ m}} = T - 0.25 \text{ kg}(9.8 \text{ m/s}^2)$$

$$T = 14.3 \text{ N}$$

c) At bottom where tension is the greatest

6. Hank rotates a bucket of mass 1.40 kg ~~is whirled~~ in a vertical circle of radius 1.10 m. At the lowest point of its motion the tension in the rope supporting the bucket is 25.0 N.

- Find the speed of the bucket.
- How fast must Hank have the bucket move at the top of the circle so that the rope does not go slack?



$m = 1.4 \text{ kg}$
 $r = 1.1 \text{ m}$
 $T_{\text{bottom}} = 25 \text{ N}$

$$a) F_c = T - F_g$$

$$\frac{mv^2}{r} = 25 \text{ N} - (1.4 \text{ kg})(9.8 \text{ m/s}^2)$$

$$\frac{mv^2}{r} = 11.28 \text{ N}$$

$$v = \sqrt{\frac{11.28(r)}{m}}$$

$$v = \sqrt{\frac{11.28(1.1 \text{ m})}{(1.4 \text{ kg})}} = 3 \text{ m/s}$$

Tension ~~is~~ 0 = before "slack"

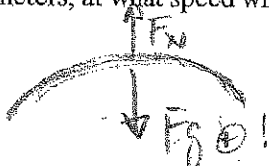
$$b) F_c = T + F_g$$

$$\frac{mv^2}{r} = mg \Rightarrow v = \sqrt{(1.1 \text{ m})(9.8 \text{ m/s}^2)}$$

$$v = \sqrt{rg} \Rightarrow v = 3.3 \text{ m/s}$$

7. Austin goes down to NYC specifically to drive over the "famous" bump on FDR Drive. If the radius of the bump is 26 meters, at what speed will Austin become airborne on this highway?

$r = 26 \text{ m}$
 $v = ?$



$$F_c = F_g - F_N$$

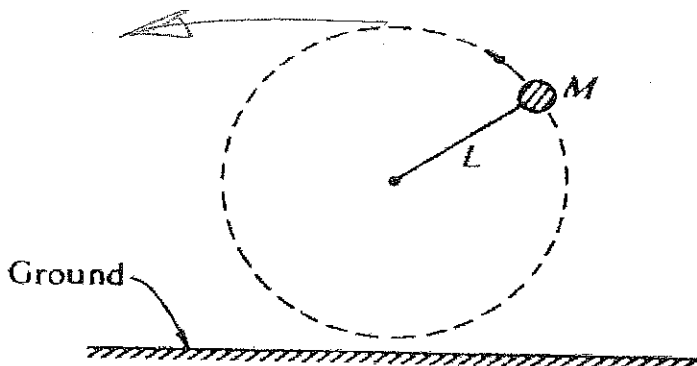
$$\frac{mv^2}{r} = -mg$$

$$v = \sqrt{gr}$$

$$v = \sqrt{(9.8 \text{ m/s}^2)(26 \text{ m})}$$

$$v = 16 \text{ m/s}$$

Sample AP Problems

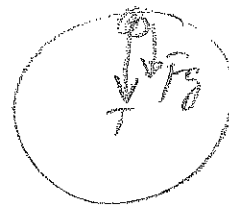


$$T = 2F_g = 2Mg$$

1. A ball of mass M attached to a string of length L moves in a circle in a vertical plane as shown above. At the top of the circular path, the tension in the string is twice the weight of the ball. At the bottom, the ball just clears the ground. Air resistance is negligible. Express all answers in terms of M , L , and g .

(a) Determine the magnitude and direction of the net force on the ball when it is at the top.

$$\begin{aligned} \Sigma F &= T + F_g \\ \Sigma F &= T + F_g \\ &= 2Mg + Mg = \boxed{3Mg} \end{aligned}$$



(b) Determine the speed v_0 of the ball at the top.

$$\begin{aligned} F_c &= T + F_g = 3Mg \\ Mv^2 &= 3Mg \\ \frac{v^2}{L} &= 3g \\ v &= \sqrt{\frac{3MgL}{M}} = \sqrt{\frac{3g}{1}} = \sqrt{3gL} \end{aligned}$$

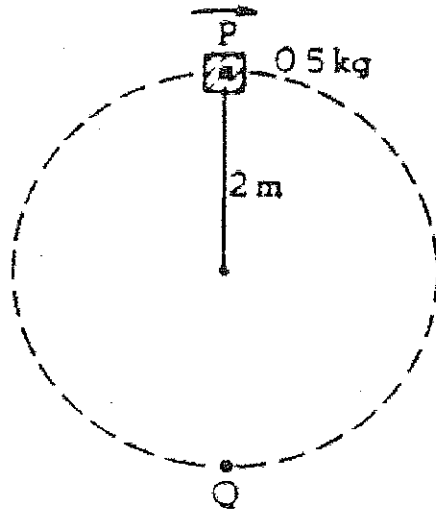
The string is then cut when the ball is at the top.

(c) Determine the time it takes the ball to reach the ground.

$$\begin{aligned} \Delta y &= v_0 t + \frac{1}{2} a t^2 \\ 2L &= \frac{1}{2} g t^2 \\ t &= \sqrt{\frac{4L}{g}} \end{aligned}$$

(d) Determine the horizontal distance the ball travels before hitting the ground.

$$\begin{aligned} d_x &= v_0 t + \frac{1}{2} a t^2 \\ d_x &= \left(\sqrt{\frac{4L}{g}}\right) \left(\sqrt{3gL}\right) = \sqrt{12L^2} \text{ or } 2\sqrt{3}L \end{aligned}$$



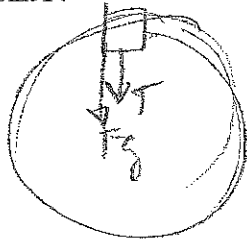
2. A 0.5-kilogram object rotates freely in a vertical circle at the end of a string of length 2 meters as shown above. As the object passes through point P at the top of the circular path, the tension in the string is 20 newtons. Assume $g = 10$ meters per second squared.

- a. Draw a free body diagram of the object and clearly label all significant forces on the object when it is at point P.

$$T = 20\text{N}$$

$$m = 0.5\text{kg}$$

$$r = 2\text{m}$$



- b. Calculate the speed of the object at point P.

$$F_c = T + F_g$$

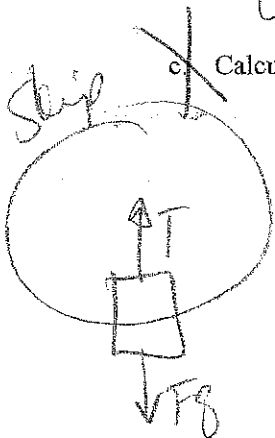
$$\frac{mv^2}{r} = T + mg$$

$$\frac{0.5\text{kg} \cdot v^2}{2\text{m}} = 20\text{N} + 5\text{N}$$

$$v = \sqrt{\frac{25(2)}{0.5}}$$

$$v = 10\text{ m/s}$$

- c. Calculate the tension in the string as the object passes through point Q.



$$F_c = T - F_g$$

$$\frac{mv^2}{r} = T - mg$$

$$\frac{(0.5\text{kg})(10\text{m/s})^2}{2\text{m}} = T - 5\text{N}$$

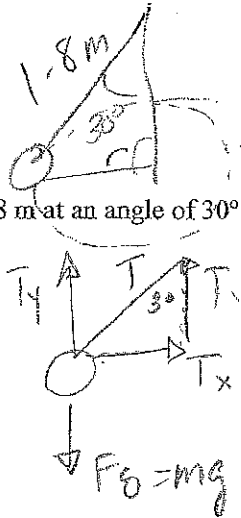
$$T = 30\text{N}$$

@ bottom
need to use
KE to solve

Conical Pendulum Problems

1. A conical pendulum has a string of length 1.8 m at an angle of 30° from the vertical. The bob has a mass of 0.3 kg. Determine:

- the tension in the string
- the linear speed of the bob
- the period of this conical pendulum



$$m = 0.3 \text{ kg}$$

$$r = \sin 30 (1.8 \text{ m}) = 0.9 \text{ m}$$

$$\sin 30 = \frac{r}{1.8 \text{ m}}$$

$$\textcircled{a} T_y = \cos 30 T$$

$$T_x = \sin 30 T$$

$$\Sigma F_y = 0$$

$$T_y - F_g = 0$$

$$T_y = F_g$$

$$\textcircled{b} \Sigma F_x = F_c$$

$$T_x = F_c$$

$$\sin 30 T = \frac{mv^2}{r}$$

$$\sin 30 (3.39 \text{ N}) = \frac{0.3 \text{ kg } v^2}{0.9 \text{ m}}$$

$$V = 2.25 \text{ m/s}$$

$$\textcircled{c} \text{ calc}$$

$$\cos 30 T = mg$$

$$0.866 T = 0.3 \text{ kg} (9.8 \text{ m/s}^2)$$

$$T = 3.39 \text{ N}$$

$$\textcircled{c} V = \frac{2\pi r}{T}$$

$$T = \frac{2\pi r}{V}$$

$$T = \frac{2\pi (0.9 \text{ m})}{2.25 \text{ m/s}} = 2.51 \text{ s}$$

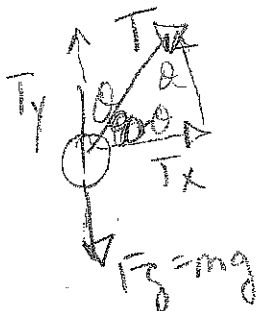
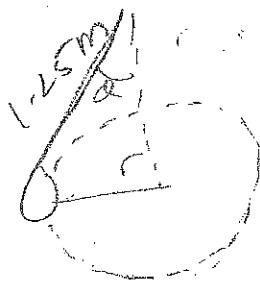
2. A conical pendulum has a bob of mass 0.1 kg and a string of length 1.25 m. Its time period is 2.00 s. Determine:

- the angle made by the string with the vertical direction
- the tension in the string

$$m = 0.1 \text{ kg}$$

$$T = 2 \text{ s}$$

$$l = 1.25$$



$$\Sigma F_y = 0$$

$$T_y - F_g = 0$$

$$T_y = F_g$$

$$T_y = mg$$

$$\cos \theta T = mg$$

$$T_x = F_c$$

$$T_x = \frac{mv^2}{r}$$

$$v = \frac{2\pi r}{T}$$

$$T_x = m \left(\frac{2\pi r}{T} \right)^2$$

$$T_x = \frac{m 4\pi^2 r^3}{T^2}$$

$$\sin \theta T = \frac{m 4\pi^2 r^3}{T^2}$$

$$T = 1623 \text{ N}$$

$$\cos \theta T = mg$$

$$\cos \theta = \frac{mg}{T}$$

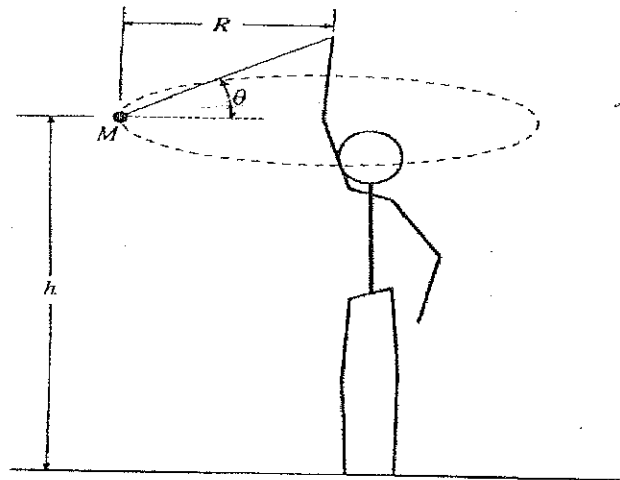
$$\theta = \cos^{-1} \frac{mg}{T}$$

$$\theta = \cos^{-1} \left(\frac{0.1 (9.8)}{1623} \right)$$

$$\theta = \cos^{-1} (0.799)$$

$$\theta = 37^\circ$$

Sample AP Problem



Skip all!

1. An object of mass M on a string is whirled with increasing speed in a horizontal circle, as shown above. When the string breaks, the object has speed v_0 , and the circular path has radius R and is a height h above the ground. Neglect air friction.

(a) Determine the following, expressing all answers in terms of h , v_0 , and g .

~~i.~~

The time required for the object to hit the ground after the string breaks

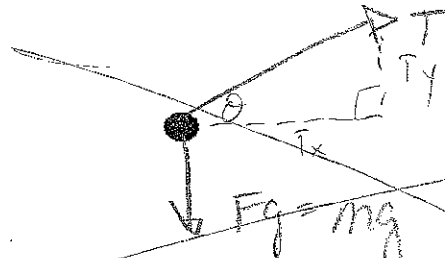
~~ii.~~

The horizontal distance the object travels from the time the string breaks until it hits the ground

~~iii.~~

The speed of the object just before it hits the ground

(b) On the figure below, draw and label all the forces acting on the object when it is in the position shown in the diagram above.



$$\begin{aligned} \sum F_y &= 0 & T_y &= mg \\ T_y - F_g &= 0 & \sin \theta T &= mg \\ T_y &= F_g & T &= mg \end{aligned}$$

same

(c) Determine the tension in the string just before the string breaks. Express your answer in terms of M , R , v_0 , and g .

~~$T = \text{max when } v = v_0!$~~

$$\begin{aligned} T_x &= F_c \\ \cos \theta T &= \frac{M v_0^2}{R} \\ T &= \frac{M v_0^2}{R \cos \theta} \end{aligned}$$

Do together?

$F_{Ny} = F_g!$

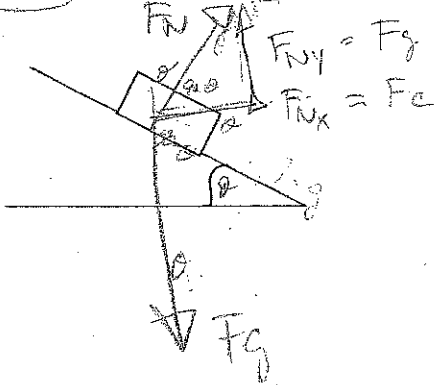
* don't break
 F_g into components!

(not $F_N = F_{gy}$?)

Just $F_N!$

Banked Curves

1. Tom likes to drive very quickly (at a velocity of v) around the Village of Tully in the rain. To protect its inhabitants, at what angle should Tully bank its curve around the village of radius, r , so that even without friction, Tom makes it safely around the village?



$F_{Nx} = \sin \theta F_N$ $F_c = \sin$

$F_{Ny} = F_g = mg$

$\cos \theta = \frac{mg}{F_N} \therefore F_N = \frac{mg}{\cos \theta}$

$F_{Nx} = \frac{mv^2}{r}$

$\sin \theta F_N = \frac{mv^2}{r}$

$\sin \theta \left(\frac{mg}{\cos \theta} \right) = \frac{mv^2}{r}$

$\tan \theta \cdot g = \frac{v^2}{rg}$

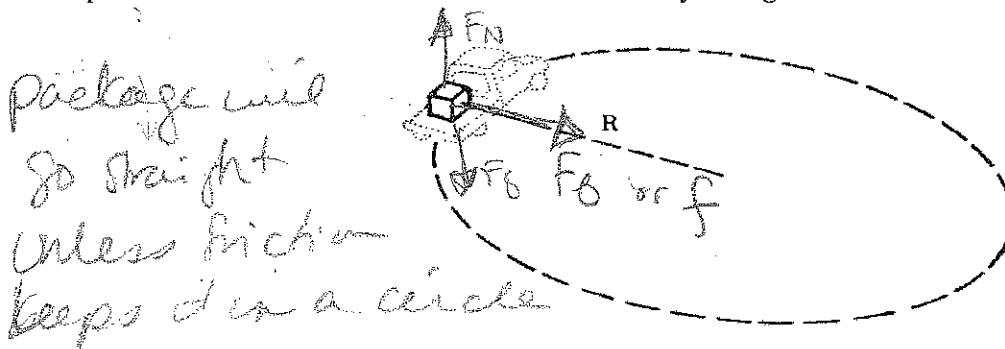
$\theta = \tan^{-1} \frac{v^2}{rg}$

$F_{Ny} = mg$
 $\cos \theta = \frac{F_{Ny}}{F_N}$

$F_N = \frac{F_{Ny}}{\cos \theta} = \frac{mg}{\cos \theta}$

Sample AP Problem

1. A box of mass M , held in place by friction, rides on the flatbed of a truck, which is traveling with constant speed v . The truck is on an unbanked circular roadway having radius of curvature R .



- (a) On the diagram provided above, indicate and clearly label all the force vectors acting on the box.
- (b) Find what condition must be satisfied by the coefficient of static friction μ between the box and the truck bed. Express your answer in terms of v , R , and g .

$$f = F_c$$

$$\mu F_N = \frac{mv^2}{r}$$

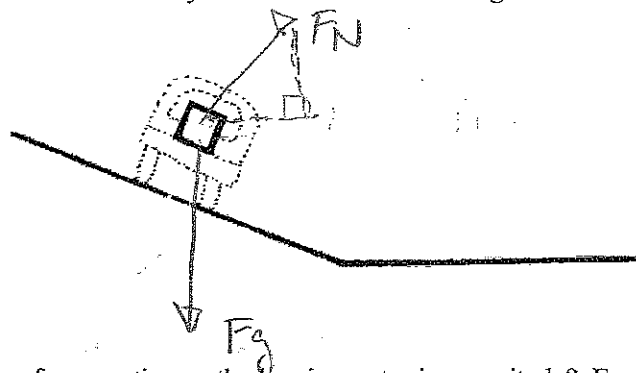
$$\mu mg = \frac{mv^2}{r}$$

$$\mu = \frac{v^2}{gr}$$

or more than

If the roadway is properly banked, the box will still remain in place on the truck for the same speed v even when the truck bed is frictionless.

- (c) On the diagram below, indicate and clearly label the two forces acting on the box under these conditions.



- (d) Which, if either, of the two forces acting on the box is greater in magnitude? Explain.

F_N must be greater because it's vertical component is equal to F_g , but its horizontal component is acting towards the center

Satellites and Orbit

$$R_E = 6.4 \times 10^6 \text{ m}$$

$$M_E = 6 \times 10^{24} \text{ kg}$$

$$= 3.5 \times 10^5 \text{ m}$$

1. What is the orbital period of a shoe lost by a careless astronaut that is located 350 km above Earth's surface?

+ Speed!

$$r = 6.4 \times 10^6 \text{ m} + 350000 = 6.75 \times 10^6 \text{ m}$$

$$T = ?$$

$$v = ?$$

$$v = \sqrt{\frac{GM_E}{r}}$$

$$= \frac{6.67 \times 10^{-11} \text{ N m}^2 / \text{kg}^2 (6 \times 10^{24} \text{ kg})}{(6.75 \times 10^6 \text{ m})^2}$$

$$= 7717 \text{ m/s}$$

$$F_g = F_c$$

$$\frac{GM_E m}{r^2} = \frac{mv^2}{r}$$

2. The space shuttle releases a satellite into a circular orbit 520 km above the Earth. How fast must the shuttle be moving (relative to Earth) when the release occurs?

Skip

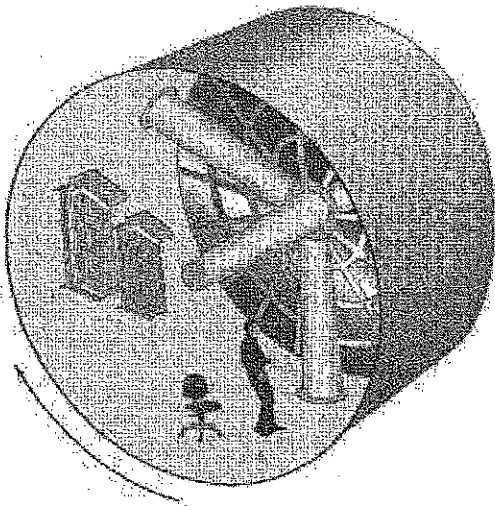
$$v = \frac{2\pi r}{T}$$

$$T \cdot 7907 \text{ m/s} = \frac{2\pi (6400350 \text{ m})}{T}$$

$$\frac{7907}{7907} = \frac{2\pi}{T} \cdot 7907$$

$$T =$$

3. At what rate must the cylindrical spaceship shown below rotate if occupants are to experience simulated gravity of 0.60g? Assume the spaceship's diameter is 50 m, and give your answer as the time needed for one revolution.



$$0.6g = (0.6 \times 9.8 \text{ m/s}^2) \quad T$$

$$F_n = F_c$$

$$m \cdot g = \frac{mv^2}{r}$$

$$(0.6 \times 9.8 \text{ m/s}^2) = \frac{v^2}{25 \text{ m}}$$

$$v = 12 \text{ m/s}$$

$$T \cdot v = \frac{2\pi r}{T} \cdot T$$

$$T = \frac{2\pi r}{v} = \frac{2\pi (25 \text{ m})}{12 \text{ m/s}}$$

$$T = 13.0 \text{ s}$$